**Lecture 12-13.**

**Local Extreme Values. The first derivative test. The second derivative test. Using the highest derivatives**

**Endpoint Extreme Values. Absolute Extreme Values.**

**Increase and decrease of functions.** The function is called *increasing* on some interval if, for any points and which belong to this interval, from the inequality we get the inequality .

The function is called *decreasing* on some interval if, for any points and which belong to this interval, from the inequality we get the inequality .

If is continuous on the interval and for then increases on the interval

If is continuous on the interval and for then decreases on the interval

In the simplest cases, the domain of definition of may be subdivided into a finite number of intervals of increase and decrease of the function ( intervals of monotonicity). These intervals are bounded by critical points (where or does not exist).

**Extremum of a function.** If there exists a two-sided neighborhood of a point such that of this neighborhood we have the inequality , then the point is called the *minimum point* *of the function*. Similarly, if for any point the inequality is fulfilled then the point is called the *maximum point of the function.* The minimum point or maximum point of a function is its *extremal point.*

If is an extremal point of the function then or does not exist (*necessary condition for the existence of an extremum*).

The *sufficient conditions* *for the existence and absence* *of an extremum* of a continuous function are given by the following rules:

1. If there exists a neighborhood of a critical point such that for and for , then is maximum point of the function and if for and for , then is minimum point of the function
2. If and , then is maximum point; if and , then is minimum point; but if

, then the point is not an extremal point of the function

More generally: let the first of the derivatives (not equal to zero at the point of the function be of the order . Then, if is even, the point is not an extremal point, namely, the maximum point if ; and it is the minimum point if But if is odd, then is not an extremal point.